SAMPLE PAPER 6: PAPER 1

QUESTION 1 (25 MARKS)

Question 1 (a)

$$4T_n = [1+(-1)^n][1+i^n]$$

$$n=1: 4T_1 = [1+(-1)^1][1+i^1] = [0][1+i] = 0 \Rightarrow T_1 = 0$$

$$n=2: 4T_2 = [1+(-1)^2][1+i^2] = [2][1-1] = [2][0] = 0 \Rightarrow T_2 = 0$$

$$n=3: 4T_3 = [1+(-1)^3][1+i^3] = [1-1][1-i] = [0][1-i] = 0 \Rightarrow T_3 = 0$$

$$n=4: 4T_4 = [1+(-1)^4][1+i^4] = [1+1][1+1] = [2][2] = 4 \Rightarrow T_4 = 1$$

0, 0, 0, 1,........

$$\therefore S_{100} = 25\times 0 + 25\times 0 + 25\times 0 + 25\times 1 = 25$$

Question 1 (b)

STEPS FOR PROOF BY INDUCTION
1. Prove result is true for some starting value of $n \in \mathbb{N}$.
2. Assume result is true for $n = k$.
3. Prove result is true for $n = (k+1)$.

1. Prove true for $n=1: 7^1 - 4^1 = 7-4 = 3$ [Therefore, true for $n = 1$.]

2. Assume true for $n=k: 7^k - 4^k = 3a,\ a \in \mathbb{N}.\ \therefore 7^k = 3a + 4^k$.

3. Prove true for $n=k+1$: Prove $7^{k+1} - 4^{k+1} = 3b,\ b \in \mathbb{N}$

Proof:

$$\begin{aligned} 7^{k+1} - 4^{k+1} &= 7(7^k) - 4^{k+1} \\ &= 7(3a+4^k) - 4^{k+1} \\ &= 21a + 7\times 4^k - 4^{k+1} \\ &= 21a + 4^k(7-4^1) \\ &= 21a + 4^k(3) \\ &= 3(7a+4^k) \\ &= 3b \end{aligned}$$

Therefore, assuming true for $n = k$ means it is true for $n = k + 1$. So true for $n = 1$ and true for $n = k$ means it is true for $n = k + 1$. This implies it is true for all $n \in \mathbb{N}$.